Test Specifications for the Massachusetts Adult Proficiency Test for Mathematics - College and Career Readiness [[1]](#footnote-1)

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**Introduction**

Since January 2003, the Center for Educational Assessment at the University of Massachusetts Amherst (UMass), under a contract awarded by the Massachusetts Department of Elementary and Secondary Education, has worked closely with the Department’s Office of Adult and Community Learning Services (ACLS) to develop achievement tests in math and reading that are appropriate for adult learners in Massachusetts. Our collaborative vision in creating the Massachusetts Adult Proficiency Test (MAPT) involved ensuring the tests are (a) aligned to the National Reporting System’s (NRS) Educational Functioning Levels (EFLs), (b) aligned with the curriculum frameworks established by ACLS and the adult basic education (ABE) community in Massachusetts, (c) sensitive enough to measure gain across the EFLs within the NRS, and (d) developed with comprehensive input from teachers and administrators from the ABE community in Massachusetts.

In early November 2013, the Massachusetts Department of Elementary and Secondary Education’s Adult and Community Learning Services (ACLS) unit announced the adoption of the College and Career Readiness Standards for Adult Education (CCRSAE; Pimentel, 2013) for use in adult basic education programs across Massachusetts. This shift from the Massachusetts Curriculum Frameworks for Mathematics and Numeracy follows implementation of the Common Core State Standards (CCSS) in K-12 education and initiatives to promote college and career readiness for all learners. With this change in curriculum, the test specifications for the MAPT will also change, and the next evolution will be the MAPT for College and Career Readiness (MAPT-CCR). The expected release date for the MAPT-CCR is July 1, 2017.

In this report, we present the final specifications to be used in developing the forthcoming MAPT for Mathematics—College and Career Readiness (MAPT for Math-CCR). Our goal in creating these test specifications was to involve ABE teachers and leaders in the field who are familiar with current mathematics standards and how they should be taught to adult learners in Massachusetts. We also sought to include mathematics specialists and those who are familiar with how mathematics is defined in the Common Core State Standards (CCSS). We accomplished these goals by convening a MAPT for Math-CCR Test Specifications *Advisory* Committee comprising three members, and a MAPT for Math-CCR Test Specifications Development Committee, comprising seven members. The Committee Members, UMass project personnel, and ACLS staff assisting in the development of the test specifications are listed in Table 1.

The process of developing these proposed test specifications took place in three phases. First, we held an in-person meeting of the Test Specifications Development Committee in June 2014. Next, we developed preliminary test specifications based on the Committee’s deliberations and sent the preliminary specifications to the Committee members for review and comment. The specifications were revised based on their feedback. Subsequently, in January 2015, the U.S. Department of Education’s Office of Career, Technical, and Adult Education (OCTAE) confirmed the NRS EFLs would retain six EFLs[[2]](#footnote-2). Given that the preliminary test specifications followed the five levels of the CCRSAE, we restructured the test specifications to be directly aligned with the six NRS EFLs, and we reconvened the Committee to review and comment on the restructured specifications during the fall of 2015. In February 2016, OCTAE released the final NRS EFLs, which are presented in Appendix A[[3]](#footnote-3).

**Table 1**

**MAPT for Math Test Specifications Development and Advisory Committees Members**

|  |  |  |
| --- | --- | --- |
| **Advisory Committee** | **Test Specifications Development Committee**  | **Affiliation** |
| Andrea (Drey) Martone | Andrea (Drey) Martone | The College of Saint Rose |
| Donna Curry | Donna Curry | Adult Numeracy Center at TERC |
| Veronica (Ronnie) Kell | Veronica (Ronnie) Kell | Mount Wachusett Community College |
|  | Barbara Goodridge | Lowell Adult Education Center |
|  | Shirley Lyon  | Northern Essex Community College |
|  | Helen McSweeney  | Berkshire County House of Correction |
|  | Michael Noonan  | Northern Essex Community College |
| **Research and Support Staff** |  |
| Stephen G. Sireci | University of Massachusetts Amherst, Center for Educational Assessment |
| April L. Zenisky |
| Joshua Marland |
| Hongyu Diao |
| Emily Pichette |
| Thompson Mechem | Massachusetts Department of Elementary & Secondary Education, Office of Adult and Community Learning Services |
| Jane Schwerdtfeger |
| Dana Varzan-Parker |

 In a previous report (Zenisky et al., 2014), we described the process we followed in working with the Committee to derive preliminary test specifications. In this report, we present the final test specifications, based on Committee deliberations during the Fall of 2015, including a conference call on October 28, 2015, and email communications through spring 2016 (to address the final EFLs). Before presenting the specifications to be used to develop the forthcoming MAPT-CCR for Mathematics, we briefly describe the steps involved in this process. For more complete details of the activities conducted before Fall 2015, see Zenisky et al. (2014).

**The MAPT for Math-CCR Test Specifications Development Committee**

To arrive at a starting point for proposing MAPT for Math-CCR test specifications, an Advisory Committee was formed prior to the June 23, 2014 meeting. The Committee reviewed (a) draft versions of the read-ahead materials, (b) the mathematics anchors associated with the CCRSAE, and (c) the mathematics foundations in those standards and on the current MAPT for Mathematics and Numeracy. The Committee emphasized that no matter what form the MAPT for Mathematics-CCR test specifications took, they must align with the CCRSAE and (consequently) with instruction practices. Based on this discussion, two-dimensional test specifications were proposed to the Test Specifications Development Committee in advance of the June meeting. The first dimension specified content, using the existing CCRSAE Domain and Conceptual Category levels.[[4]](#footnote-4) The second dimension concerned cognitive levels and was not defined by the advisory committee (this task was left to the full specifications committee).

**Description of the June 2014 Meeting**

 The Committee members and UMass and ACLS staff met for a full-day at the University of Massachusetts (UMass) on June 23, 2014. All participants read the read-ahead materials in advance and were prepared to provide their input on the proposed test specifications. The purpose of the meeting was to come up with proposed test specifications for the MAPT for Math -CCR. The Committee used several documents to influence the development of these test specifications, including:

* College and Career Readiness Standards for Adult Education (CCRSAE)
* The current MAPT for Mathematics and Numeracy test specifications, and
* Specifications for newer tests “aligned” with CCRSAE

In addition, the Committee was encouraged to use their instructional and assessment experiences to facilitate alignment of the MAPT for Mathematics-CCR to instruction in ABE classrooms.

 The read-ahead materials for the Committee meeting are published as Zenisky and Sireci (2014). These materials included the CCRSAE, and the test specifications for newer tests aligned with the CCRSAE. The following assessments were deemed relevant to the MAPT for Mathematics-CCR:

* the Partnership for the Assessment of Readiness for College and Careers (PARCC)
* the Smarter Balanced Assessment Consortium
* the HiSET high school equivalency exam
* the GED high school equivalency exam, and
* the TASC high school equivalency exam.

The specifications for these assessments were included in the read-ahead materials (Zenisky & Sireci, 2014).

Before discussing the read-ahead materials, a brief overview of the history of the MAPT, and of the current and evolving Federal regulations governing assessment in ABE were discussed. Following this discussion, we moved to a review of the test specification dimensions that were proposed for the MAPT for Mathematics-CCR. Two dimensions were proposed—one delineating “Content Topics,” the other delineating “Cognitive Levels.”

**Aligning the Test Specifications with the CCRSAE**

In advance of considering the test specifications for the MAPT-CCR, we reflected on the CCRSAE standards and how it is conceptualized. As described in the read-ahead materials, Pimentel (2013) arranged the CCRSAE in five grade-level groupings. These grade level groupings differ from the “Educational Functioning Levels” (EFLs) currently measured on the MAPT. The National Reporting System developed by the U.S. Department of Education specified six EFLs; the current MAPT measures the top five of those six EFLs, as will the MAPT-CCR. The correspondence between the CCRSAE and the current MAPT test levels is provided in Table 2. The Pimentel (2013) structure places some of the grade level equivalent skills from 6th grade in the CCR “C” level and others in the CCR “D” level, whereas the MAPT Level 4 (High Intermediate) comprises all of 6th grade.

**Table 2**

**Correspondence between CCRSAE levels and Current MAPT Test Levels**

|  |  |
| --- | --- |
| **College and Career Readiness Standards for Adult Education (Pimentel 2013)\*** | **Current MAPT Test Levels** **(Educational Functioning Levels)** |
| CCR A: K-1 Beginning Adult Basic Education |  |
| CCR B: 2-3Beginning Basic Education | MAPT Level 2: GLE 2-3.9Beginning Basic Education |
| CCR C: 4-5, 6 Low Intermediate Basic Education | MAPT Level 3: GLE 4-5.9 Low Intermediate Basic Education |
| CCR D: 6, 7-8 High Intermediate Basic Education | MAPT Level 4: GLE 6-8.9High Intermediate Basic Education  |
| CCR E: High School Low Adult Secondary and High Adult Secondary Education | MAPT Level 5: GLE 9-10.9Low Adult Secondary |
| MAPT Level 6: GLE 11-12.9High Adult Secondary |

The Committee discussed the merits of setting test specifications using both the CCRSAE and the NRS EFLs, and it was determined that although there are differences across the CCRSAE and EFLs, there is also compatibility. After OCTAE released the proposed EFLs in January 2015, it was decided the MAPT-CCR should have five levels linked to EFLs 2 (Beginning Basic) through 6 (High Adult Secondary). The final NRS *Implementation Guidelines* that were released in February 2016 (OVAE, 2016) described the relationship between the CCRSAE and EFLs (p. E11). In Table 3, we reflect this relationship by indicating the correspondence between the CCRSAE and the MAPT for Math-CCR. This table indicates where the standards (benchmarks) from the CCRSAE will be drawn for the MAPT for Math-CCR levels that correspond to the 2016 NRS EFLs.

**Table 3**

**Correspondence between NRS EFLs, MAPT for Math-CCR Levels and CCRSAE**

|  |  |  |
| --- | --- | --- |
| NRS EFL | MAPT-CCR Level | Standards *primarily* from CCRSAE Level |
| Beginning Basic | 2 | B |
| Low Intermediate | 3 | C |
| Middle Intermediate | 4 | D |
| High Intermediate | 5 | D |
| Adult Secondary | 6 | E |

**Content Alignment with the CCRSAE**

To facilitate the discussion around the issue of aligning the MAPT-CCR with the CCRSAE, the Committee reviewed the structure of the CCRSAE for Mathematics and we re-emphasized that a goal in developing the tests was to ensure that they are aligned with the standards to support ABE instruction. The CCRSAE Standards for Mathematical Content document defines the domain of ABE mathematics using several dimensions. For Levels B, C, and D, the first of these is Domains, as shown in Table 4. At Level E, a different structure was used in the CCRSAE document, and this approach itself is drawn from the CCSS. Here, at Level E, content is first defined into Conceptual Categories and then Domains are assigned to categories (Table 5).

In reflecting on the domains and conceptual category structure in Tables 4 and 5, the Committee noted the domain structure could be an appropriate way of building content. After some discussion, the committee came to consensus on the content groupings of domains that are presented in Table 6.

**Table 4**

**CCRSAE Content Domains by CCRSAE Levels (B, C, and D only)**

|  |  |
| --- | --- |
| **CCRSAE Level**  | **Content Domains** |
| B: 2-3 | Numbers and Operations: Base TenNumbers and Operations: FractionsOperations and Algebraic ThinkingGeometryMeasurement and Data |
| C: 4-5, 6 | Numbers and Operations: Base TenThe Number System Numbers and Operations: FractionsRatios and Proportional RelationshipsOperations and Algebraic ThinkingExpressions and EquationsGeometryMeasurement and DataStatistics and Probability |
| D: 6, 7-8 | The Number System Expressions and EquationsFunctionsGeometryStatistics and Probability |

**Table 5**

**CCRSAE Content Domains by CCRSAE Level (E only)**

|  |  |  |
| --- | --- | --- |
| **CCRSAE Level**  | **Conceptual Categories** | **Domains**  |
| E: High School  | Number and Quantity | The Real Number SystemQuantity  |
| Algebra | Seeing Structure in ExpressionsArithmetic with Polynomials and Rational ExpressionsCreating EquationsReasoning with Equations and Inequalities |
| Functions | Interpreting FunctionsBuilding Functions Linear, Quadratic, and Exponential Models |
| Geometry | CongruenceSimilarity, Right Triangles, and TrigonometryGeometric Measurement and DimensionModeling with Geometry |
| Statistics and Probability | Interpreting Categorical and Quantitative Data |

**Table 6**

**Committee Recommendation on Condensed Content Domains (All Levels)**

|  |  |
| --- | --- |
| **Domain Groups** | **Level** |
| **B** | **C** | **D** | **E** |
| Numbers and Operations: Base Ten | Numbers and Operations: Base Ten | Numbers and Operations: Base Ten |  |  |
| Operations and Algebraic Thinking & Expressions and Equations | Operations and Algebraic Thinking | Operations and Algebraic Thinking and Expressions and Equations | Expressions and Equations | Algebra |
| Numbers and Operations: Fractions & Ratios and Proportional Relationships | Numbers and Operations: Fractions | Numbers and Operations: Fractions and Ratios and Proportional Relationships |  |  |
| The Number System |  | The Number System | The Number System | Numbers and Quantity |
| Geometry | Geometry | Geometry | Geometry | Geometry |
| Measurement and Data & Statistics and Probability | Measurement and Data | Measurement and Data and Statistics and Probability | Statistics and Probability | Statistics and Probability |
| Functions |  |  | Functions | Functions |

This approach resulted in five domain groups at Level B, six domain groups at Level C, five domain groups at Level D, and five domain groups at Level E. The Committee did not suggest any reorganization of standards here, but rather suggested a minor grouping of existing domain labels to facilitate consistency of the content specification structure across levels, and still allow for the flexibility for different content elements to be represented more or less heavily depending on the balance of domains, cluster headings, and standards in each curricular level.

**Developing Cognitive Levels**

The Committee discussed whether the test specifications should include a cognitive dimension. The current MAPT has used a modified Bloom’s taxonomy for representing the cognitive complexity of MAPT items, with three levels defined as a) Knowledge & Comprehension, b) Application, and c) Analysis, Synthesis, Evaluation (Sireci, et al., 2004). The Committee strongly recommended the use of a cognitive dimension for the MAPT-CCR for Mathematics.

The choice of how to conceptualize this cognitive dimension was discussed at length by the Committee. They reflected on the various representations of cognitive category used by other relevant tests, including PARCC, Smarter Balanced, HiSET, GED, and TASC. The PARCC structure of subclaims (with different types of tasks associated with each subclaim) was remarked on by the Committee as being particularly relevant to an assessment of the CCRSAE, but was noted as involving performance assessment, which is not feasible with the present computer-adaptive MAPT.

Part of the concerns expressed by the Committee here was also how to reflect and prioritize the *Standards for Mathematical Practices*, as these were explained to be a critical component of both the CCSS and the CCRSAE. It was noted that perhaps even more than the content organization described in the CCSS and CCRSAE, the spirit of the changes in the curricula for mathematics instructors lies in these Practices, which themselves entail a considerable shifting of what it means to think mathematically both as a teacher and a learner. These Mathematical Practice standards are listed below.

* Standard 1: Make sense of problems and persevere in solving them
* Standard 2: Reason abstractly and quantitatively
* Standard 3: Construct viable arguments and critique the reasoning of others
* Standard 4: Model with mathematics
* Standard 5: Use appropriate tools strategically
* Standard 6: Attend to precision
* Standard 7: Look for and make use of structure
* Standard 8: Look for and express regularity in repeated reasoning

In light of these Standards, the Committee discussed three main frameworks for conceptualizing cognitive areas and chose to merge elements of each approach to create the following three cognitive components for the MAPT for Math-CCR:

* Recall (Procedural Understanding)
* Skill / Concept (Conceptual Understanding)
* Strategic Thinking (Problem Solving)

The definitions for these components are based on the text that makes up the Components of Rigor, as follows (<http://www.p12.nysed.gov/ciai/mst/math/standards/revisedlintro.html>).

Recall (Procedural Understanding) is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). Although the word procedural may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics, such as measuring the size of an angle using a protractor. The use of calculators need not threaten the development of students’ computational skills. On the contrary, calculators can enhance both understanding and computing if used properly and effectively. Accuracy and efficiency with procedures are important, but they should be developed through understanding. When students learn procedures through understanding, they are more likely to remember the procedures and less likely to make common computational errors.

Skill / Concept (Conceptual Understanding) consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Strategic Thinking (Problem Solving) is the ability to formulate, represent, and solve mathematical problems. Problems generally fall into three types:

* one-step problems
* multi-step problems
* process problems

Most problems that students will encounter in the real world are multi-step or process problems. Solution of these problems involves the integration of conceptual understanding and procedural knowledge. Students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution. Therefore, mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself is not useful in problem solving unless one recognizes when and where to use it as well as when and where it does not apply. Many textbook problems are not typical of those that students will meet in real life. Therefore, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

**Score Reporting and the Current MAPT**

In addition to the current MAPT for Mathematics test specifications, specifications for other assessments, and the CCRSAE, we also discussed the current MAPT for Mathematics score reports with the Committee. The Committee agreed that the test specifications should consider and support how results will be reported at both the individual student and classroom levels.

**Proposed Revised Test Specifications**

After discussing the preliminary report from the June 2014 meeting, the Test Specifications Development Committee unanimously agreed that the Domain Groups and the Components of Rigor would provide a good framework for specifying the knowledge and skills to be measured on the MAPT for Mathematics-CCR in a way that would align them with the CCRSAE and with future instructional changes in ABE classes in Massachusetts.

**Specifications Review Process**

 After confirming the dimensions to be used as the framework for the test specifications, the Committee reviewed each test level. The intended process to be followed by the Committee was to review each test level and complete the following tasks:

1. determine whether any Domain Groups should be added or removed
2. determine the percentages to be allotted to each Domain Group
3. determine the percentages to be allotted to the Components of Rigor Cognitive Levels, and
4. ensure the percentages were as reflective as possible of the knowledge and skills defined in the NRS EFLs.

The results of the Committee members’ reviews and discussions (in 2014, 2015, and 2016) are presented in Tables 7 through 11. These tables represent the final test specifications for the MAPT for Math-CCR. These test specifications reflect the relative emphases of the Domain Groups in the CCRSAE and the knowledge and skills described at each NRS EFL. They also reflect an increase in cognitive complexity with “Recall” decreasing after Level 3, relative to “Strategic Thinking.” ThThis slight shift in cognitive components from Levels 2 through 6 reflects the Committee’s preference for general consistency of cognitive components across the test levels, in accordance with the spirit of the Common Core to promote a range of thinking across the full range of proficiency.

Table 7

Test Specifications for MAPT-CCR Level 2 (EFL Beginning Basic, GLE 2-3)

|  |  |  |
| --- | --- | --- |
| **Domain Groups** | **Cognitive Components**  | **Total (%)** |
| **Recall**  | **Skill / Concept**  | **Strategic Thinking** |
| Numbers and Operations: Base Ten (NBT) | 7.5%-10% | 7.5%-10% | 5%-7.5% | 25% |
| Operations and Algebraic Thinking (OA)  | 7.5%-10% | 7.5%-10% | 5%-7.5% | 25% |
| Numbers and Operations: Fractions (NF)  | 2.5%-5% | 2.5%-5% | 2.5%-5% | 10% |
| The Number System (NS) |  |  |  |  |
| Geometry (G) | 2.5%-5% | 2.5%-5% | 2.5%-5% | 10% |
| Measurement and Data (MD) | 10%-12.5% | 10%-12.5% | 7.5%-10% | 30% |
| Functions (F) |  |  |  |  |
| Total (%) | 35% | 35% | 30% | 100% |

Table 8

Specifications for MAPT for Math-CCR Level 3 (EFL Low Intermediate, GLE 4-5)

|  |  |  |
| --- | --- | --- |
| Domain Groups | Cognitive Components  | Total (%) |
| Recall  | Skill / Concept  | Strategic Thinking |
| Numbers and Operations: Base Ten (NBT) | 5%-7.5% | 5%-7.5% | 2.5%-5% | 15% |
| Operations and Algebraic Thinking (OA) and Expressions and Equations (EE) | 7.5%-10% | 7.5%-10% | 5%-7.5% | 25% |
| Numbers and Operations: Fractions (NF) and Ratios and Proportional Relationships (RP) | 7.5%-10% | 7.5%-10% | 5%-7.5% | 25% |
| The Number System (NS) | 0%-2.5% | 0%-2.5% | 0%-2.5% | 5% |
| Geometry (G) | 2.5%-5% | 2.5%-5% | 2.5%-5% | 10% |
| Measurement and Data (MD) and Statistics and Probability (SP) | 5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Functions (F) |  |  |  |  |
| Total | 35% | 35% | 30% | 100% |

Table 9

Specifications for MAPT for Math-CCR Level 4 (EFL Middle Intermediate, GLE 6-7)

|  |  |  |
| --- | --- | --- |
| **Domain Groups** | **Cognitive Components**  | **Total (%)** |
| **Recall**  | **Skill / Concept**  | **Strategic Thinking** |
| Numbers and Operations: Base Ten (NBT) |  |  |  |  |
| Expressions and Equations (EE) | 5%-7.5% | 7.5%-10% | 7.5%-10% | 25% |
| Ratios and Proportional Relationships (RP) | 2.5%-5% | 2.5%-5% | 2.5%-5% | 10% |
| The Number System (NS) | 5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Geometry (G) | 2.5%-5% | 5%-7.5% | 5%-7.5% | 15% |
| Statistics and Probability (SP) | 5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Functions (F) | 2.5%-5% | 2.5%-5% | 2.5%-5% | 10% |
| Total | 30% | 35% | 35% | 100% |

Table 10

Specifications for MAPT for Math-CCR Level 5 (EFL High Intermediate, GLE 8-9)

|  |  |  |
| --- | --- | --- |
| **Domain Groups** | **Cognitive Components**  | **Total (%)** |
| **Recall**  | **Skill / Concept**  | **Strategic Thinking** |
| Numbers and Operations: Base Ten (NBT) |  |  |  |  |
| Expressions and Equations (EE) | 7.5%-10% | 10%-12.5% | 10%-12.5% | 30% |
| Ratios and Proportional Relationships (RP) |  |  |  |  |
| The Number System (NS) | 2.5%-5% | 5%-7.5% | 5%-7.5% | 15% |
| Geometry (G) | 5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Statistics and Probability (SP) | 2.5%-5% | 7.5%-10% | 7.5%-10% | 20% |
| Functions (F) | 2.5%-7.5% | 5%-7.5% | 5%-7.5% | 15% |
| Total | 30% | 35% | 35% | 100% |

Table 11

Specifications for MAPT for Math-CCR Level 6 (EFL Adult Secondary, GLE 9-12.9)

|  |  |  |
| --- | --- | --- |
| **Domain Groups** | **Cognitive Components**  | **Total (%)** |
| **Recall**  | **Skill / Concept**  | **Strategic Thinking** |
| Numbers and Operations: Base Ten (NBT) |  |  |  |  |
| Expressions and Equations (EE) | 7.5%-10% | 10%-15% | 10%-15% | 35% |
| Ratios and Proportional Relationships (RP) |  |  |  |  |
| The Number System (NS) | 2.5%-5% | 2.5%-7.5% | 2.5%-7.5% | 10% |
| Geometry (G) | 2.5%-7.5% | 2.5%-7.5% | 2.5%-7.5% | 15% |
| Statistics and Probability (SP) | 2.5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Functions (F) | 5%-7.5% | 5%-7.5% | 5%-7.5% | 20% |
| Total | 25% | 35% | 40% | 100% |

**Summary**

The overall proportions within and across cells shown in Tables 7 through 11 will guide the MAPT for Math-CCR test developers in the task of assembling test forms that assess what ABE learners in Massachusetts know and can do relative to the CCRSAE, prioritizing the balance of content and cognitive skills laid out in the curriculum at each level, and will also reflect the knowledge and skills described in the 2016 NRS EFLs. These test specifications will guide development of items, and “panels” (similar to test forms in multistage-adaptive testing) for these new tests. These specifications are based on extensive input from mathematics experts and ABE teachers throughout Massachusetts and beyond, and they clearly define what is measured on the MAPT for Mathematics-CCR.

**References**

Pimentel, S. (2013). College and career readiness standards for adult education. Washington, DC: U.S. Department of Education, Office of Vocational and Adult Education. Available: <http://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf>

Sireci, S. G., Baldwin, P., Keller, L. A., Valle, M., Goodridge, B., Leonelli, E., Schmitt, M. J., Tamarkin, K., & Titzel, J. (2004). Specifications for the ACLS Mathematics and Numeracy Proficiency Tests. *Center for Educational Assessment Research Report No. 513*. Amherst, MA: School of Education, University of Massachusetts.

Zenisky, A. L., & Sireci, S. G. (2014). Developing specifications for the Massachusetts Adult Proficiency Test for Mathematics -College and Career Readiness. *Center for Educational Assessment Research Report No. 877*. Amherst, MA: College of Education, University of Massachusetts.

Zenisky, A. L., & Sireci, S. G., Diao, H., Curry, D., Goodridge, B., Kell, V., Lyon, S., Martone, A., McSweeney, H., Noonan, M., & Schwerdtfeger, J. (2014, December). Test Specifications for the Massachusetts Adult Proficiency Test for Mathematics - College and Career Readiness. *Center for Educational Assessment Research Report No. 889*. Amherst, MA: College of Education, University of Massachusetts.

**Appendix A**

**Educational Functioning Level Descriptors: Mathematics[[5]](#footnote-5)**

**Introduction and Process**

The Educational Functioning Level (EFL) Descriptors for Mathematics are written using the College and Career Readiness Standards for Adult Education (CCR) as the foundation. They are intended to guide both teaching and assessment for adult learners. While these narrative descriptors address the most critical concepts for adult learners (as defined in the Major Work of the Level), there are additional concepts found in the CCR standards that support the major work for each level, and that are included in these descriptors. Lesson plans and assessment items for adult learners should be based on the full text of the CCR standards for each level, using these critical concepts as the foundation for lesson development and assessment.

The mathematics descriptors are divided into six educational functioning levels. The levels are Beginning Literacy (corresponding to Level A of the CCR); Beginning Basic (corresponding to Level B of the CCR); Low Intermediate (corresponding to Level C of the CCR); Middle Intermediate (corresponding to part of the Level D CCR), High Intermediate (corresponding to the remainder of the Level D CCR); and Adult Secondary (corresponding to Level E of the CCR). Each of the levels corresponds roughly to two grade levels, in K-12 terms, except for Level E, which combines the critical concepts of all of grades 9 through 12. Within each level the descriptors are further divided by domain: *The Mathematical Practices, Number Sense and Operations, Algebraic Thinking, Geometry (and Measurement), and Data Analysis (Statistics and Probability).*

The descriptors do not provide a complete or comprehensive delineation of all of the skills at any given level but provide examples of the most critical concepts and skills for the level to guide assessment and instruction. Assessment of the Mathematical Practice descriptors are best performed in the classroom using assessments that could be formative or summative and may be informal. It should be noted that mathematics placement decisions should take into account the reading level of the adult student. Verbally presented application problems at all mathematics levels require a minimum reading level.

**Level 1: Beginning Literacy**

*The Mathematical Practices*: Students prepared to exit this level are able to decipher a simple problem presented in a context and reason about and apply correct units to the results. They can visualize a situation using manipulatives or drawings and explain their processes and results using mathematical terms and symbols appropriate for the level. They recognize errors in the work and reasoning of others. They are able to strategically select and use appropriate tools to aid in their work, such as pencil/paper, measuring devices, and/or manipulatives. They can see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

*Number Sense and Operations*: Students prepared to exit this level have an understanding of whole number place value for tens and ones and are able to use their understanding of place value to compare two-digit numbers. They are able to add whole numbers within 100 and explain their reasoning, e.g., using concrete models or drawings and strategies based on place value and/or properties of operations. They are able to apply their knowledge of whole number addition and subtraction to represent and solve word problems that call for addition of three whole numbers whose sum is less than 20 by using such problem-solving tools as objects, drawings, and/or simple equations.

*Algebraic Thinking:* Students prepared to exit this level understand and apply the properties of operations to addition and subtraction problems. They understand the relationship between the two operations and can determine the unknown number in addition or subtraction equations.

*Geometry and Measurement*: Students prepared to exit this level can analyze and compare 2-dimensional and 3-dimensional shapes based on their attributes, such as their shape, size, orientation, the number of sides and/or vertices (angles), or the lengths of their sides. They can reason with two-dimensional shapes (e.g., quadrilaterals and half- and quarter-circles) and with three-dimensional shapes (e.g., right prisms, cones, and cylinders) to create composite shapes. They are able to measure the length of an object as a whole number of units, which are not necessarily standard units, for example measuring the length of a pencil using a paper clip as the length unit.

*Data Analysis*: Students prepared to exit this level are able to organize, represent, and interpret simple data sets (e.g., lists of numbers, shapes, or items) using up to three categories. They can answer basic questions related to the total number of data points in a set and the number of data points in each category, and can compare the number of data points in the different categories.

**Level 2:** **Beginning Basic**

*The Mathematical Practices*: Students prepared to exit this level are able to decipher two-step problems presented in a context, visualizing a situation using diagrams or sketches, and reasoning about and applying the correct units and the proper degree of precision to the results. They can explain their processes and results using mathematical terms and symbols appropriate for the level and recognize errors in the reasoning of others. They strategically select and use the appropriate tools to aid in their work, such as pencil/paper, measuring devices, manipulatives, and/or calculators. They are able to see patterns and structure in sets of numbers, including in multiplication or addition tables, and use those insights to work more efficiently.

*Number Sense and Operations*: Students prepared to exit this level understand place value for whole numbers to 1000 and can use that understanding to read, write, count, compare, and round three-digit whole numbers to the nearest 10 or 100. They are able to compute fluently with all four operations with whole numbers within 100. They use place value and properties of operations to explain why addition and subtraction strategies work, and can demonstrate an understanding of the inverse relationship between multiplication and division. They can solve one- and two-step word problems involving all four operations within 100 and identify and explain arithmetic patterns. They have an understanding of fractions, especially unit fractions, and can represent simple fractions on a number line. They understand and can explain equivalence of fractions, can recognize and generate simple equivalent fractions, and can compare two fractions with the same numerator or denominator by reasoning about their size.

*Algebraic Thinking:* Students prepared to exit this level apply the properties of operations to multiplication and division of whole numbers. They understand the relationship between multiplication and division and can determine the unknown number in multiplication or division equations.

*Geometry and Measurement*: Students prepared to exit this level are able to reason about geometric shapes and their attributes. They can demonstrate an understanding that different shapes might share common attributes (e.g., four sides) and can compare and classify two-dimensional shapes, particularly quadrilaterals. They are able to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. They can use common U.S. Customary and metric units for linear measurements (e.g., inches, feet, centimeters, and meters) and solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. They understand the concept of area and can relate it to addition and multiplication to solve real-world problems. They also understand, and can solve, real-world and mathematical problems involving perimeter of polygons.

*Data Analysis*: Students prepared to exit this level are able to draw and interpret simple graphs (e.g., bar graphs, picture graphs, and number line diagrams) including scaled bar and picture graphs. They can solve one- and two-step problems using scaled bar graphs. They can generate measurement data by measuring lengths to the nearest half- and quarter-inch and display that data by making a line plot marked off in appropriate units.

**Level 3:** **Low Intermediate**

*The Mathematical Practices*: Students prepared to exit this level are able to decipher multi-step problems presented in a context and reason about and apply the correct units and the proper degree of precision to the results. They can visualize a situation using diagrams or sketches, see multiple strategies for solving a problem, explain their processes and results, and recognize errors in the work and reasoning of others. They can express themselves using mathematical terms and notation appropriate for the level and can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, and/or technology. They are able to see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

*Number Sense and Operations*: Students prepared to exit this level understand place value for both multi-digit whole numbers and decimals to thousandths, and use their understanding to read, write, compare, and round decimals. They are able to use their place value understanding and properties of operations to fluently perform operations with multi-digit whole numbers and decimals. They can find common factors, common multiples, and understand fraction concepts, including fraction equivalence and comparison. They can add, subtract, multiply and divide with fractions and mixed numbers. They are able to solve multi-step word problems posed with whole numbers and fractions, using the four operations. They also have an understanding of ratio concepts and can use ratio language to describe a relationship between two quantities, including the concept of a unit rate associated with a ratio.

*Algebraic Thinking:* Students prepared to exit this level are able to apply and extend their understanding of arithmetic to algebraic expressions, using a symbol to represent an unknown value. They can write, evaluate, and interpret expressions and equations, including expressions that arise from formulas used in real-world problems. They can solve real-world and mathematical problems by writing and solving simple one-variable equations and write a simple inequality that represents a constraint or condition in a real-world or mathematical problem. They can represent and analyze quantitative relationships between dependent and independent variables.

*Geometry and Measurement*: Students prepared to exit this level have a basic understanding of the coordinate plane and can plot points (i.e., ordered pairs) and place polygons in the coordinate plane to solve real-world and mathematical problems. They can classify two-dimensional shapes and use formulas to determine the area of two-dimensional shapes such as triangles and quadrilaterals. They can determine the surface area of three-dimensional shapes composed of rectangles and triangles, and find the volume of right rectangular prisms. They are able to convert like measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions to solve multi-step, real-world problems. They are also able to solve measurement word problems (such as those that involve area, perimeter, distance, time intervals, liquid volumes, mass, and money) that involve simple fractions or decimals.

*Data Analysis and Statistics*: Students prepared to exit this level have a basic conceptual understanding of statistical variability, including such concepts as center, spread, and the overall shape of a distribution of data. They can present data using displays such as dot plots, histograms, and box plots.

**Level 4: Middle Intermediate**

*The Mathematical Practices*: Students prepared to exit this level are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems. They can express themselves using the mathematical terms and notation appropriate to the level. They are able to defend their findings and critique the reasoning of others. They are accurate in their calculations and use estimation strategies to assess the reasonableness of their results. They can create algebraic and geometric models and use them to answer questions and solve problems. They can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, calculators, and/or spreadsheets. They are able to see patterns and structure in number sets, data, expressions and equations, and geometric figures.

*Number Sense and Operations*: Students prepared to exit this level have an understanding of the rational number system, including how rational numbers can be represented on a number line and pairs of rational numbers can be represented on a coordinate plane. They can apply the concept of absolute value to find horizontal and vertical distances. They are able to apply the properties of integer exponents and evaluate, estimate, and compare simple square roots and cube roots. Individuals at this level also understand ratio, rate, and percent concepts, as well as proportional relationships.

*Algebraic Thinking:* Students prepared to exit this level understand the connections between proportional relationships, lines, and linear equations. They understand numerical and algebraic expressions, and equations and are able to use them to solve real-world and mathematical problems. They are able to analyze and solve linear equations and pairs of simultaneous linear equations. Individuals at this level are able to define, interpret, and compare linear functions.

*Geometry*: Students prepared to exit this level can solve real-world and mathematical problems that involve angle measure, circumference, and area of 2-dimensional figures. They are able to solve problems involving scale drawings of 2-dimensional geometric figures. They understand the concepts of congruence and similarity with respect to 2-dimensional figures. They understand the Pythagorean theorem and can apply it to determine missing lengths in right triangles.

*Statistics and Probability*: Students prepared to exit this level can summarize and describe numerical data sets in relation to their context, including determining measures of center and variability and describing patterns and/or striking deviations from patterns. They understand and can apply the concept of chance, or probability. They are able to use scatter plots for bivariate measurement data to describe patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association).

**Level 5: High Intermediate**

*The Mathematical Practices*: Students prepared to exit this level are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems. They can reason quantitatively, including using units as a way to solve problems. They are able to defend their findings and critique the reasoning of others. They are accurate in their calculations and use estimation strategies to assess the reasonableness of their results. They can create algebraic and geometric models and use them to answer questions and solve problems. They can strategically select and use tools to aid in their work, such as graphing calculators, spreadsheets, and/or computer software. They are able to make generalizations based on patterns and structure they discover in number sets, data, expressions and equations, and geometric figures and use these insights to work more efficiently.

*Number Sense and Operations*: Students prepared to exit this level can reason about and solve real-world and mathematical problems that involve the four operations with rational numbers. They can apply the concept of absolute value to demonstrate on a number line their understanding of addition and subtraction with negative and positive rational numbers. Individuals at this level can apply ratio and percent concepts, including using rates and proportional relationships to solve multi-step real-world and mathematical problems.

*Algebraic Thinking:* Students prepared to exit this level are able to use algebraic and graphical representations to solve real-world and mathematical problems, involving linear equations, inequalities, and pairs of simultaneous linear equations. Individuals at this level are able to use linear functions to describe, analyze, and model linear relationships between quantities.

*Geometry*: Students prepared to exit this level can solve real-world and mathematical problems that involve volume and surface area of 3-dimensional geometric figures. They can use informal arguments to establish facts about various angle relationships such as the relationships between angles created when parallel lines are cut by a transversal. They apply the Pythagorean theorem to determine lengths in real-world contexts and distances in the coordinate plane.

*Statistics and Probability*: Students prepared to exit this level can use random sampling to draw inferences about a population and are able to draw informal comparative inferences about two populations using measures of center and measures of variability for numerical data from random samples. They can develop, use, and evaluate probability models. They are able to use scatter plots for bivariate measurement data to interpret patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association) and a 2-way table to summarize and interpret bivariate categorical data.

**Level 6: Adult Secondary**

*The Mathematical Practices*: Students prepared to exit this level are able to think critically, make assumptions based on a situation, select an efficient strategy from multiple possible problem-solving strategies, plan a solution pathway, and make adjustments as needed when solving problems. They persevere in solving challenging problems, including considering analogous, simpler problems as a way to solving a more complex one. They can reason quantitatively, including through the use of units, and can express themselves using the precise definitions and mathematical terms and notation appropriate to the level. They are accurate in their calculations, use an appropriate level of precision in finding solutions and reporting results, and use estimation strategies to assess the reasonableness of their results. They are able to make conjectures, use logic to defend their conclusions, and can detect faulty thinking and errors caused by improper use of technology. They can create algebraic and geometric models and use them to answer questions, interpret data, make predictions, and solve problems. They can create algebraic and geometric models and use them to answer questions, interpret data, make predictions, and solve problems. They can strategically select and use tools, such as measuring devices, calculators, spreadsheets, and/or computer software, to aid in their work. They are able to see patterns and structure in calculations, expressions, and equations and make connections to algebraic generalizations, which they use to work more efficiently.

*Number Sense and Operations:* Students prepared to exit this level have extended their number sense to include irrational numbers, radicals, and rational exponents and understand and use the set of real numbers. They are able to assess the reasonableness of calculation results based on the limitations of technology or given units and quantities and give results with the appropriate degree of precision.

*Algebraic Thinking:* Students prepared to exit this level understand the structure of expressions and can use that structure to rewrite linear, exponential, and quadratic expressions. They can add, subtract, and multiply polynomials that involve linear and/or quadratic expressions. They are also able to create linear equations and inequalities and quadratic and simple exponential equations to represent relationships between quantities and can represent constraints by linear equations or inequalities, or by systems of linear equations and/or inequalities. They can interpret the structure of polynomial and rational expressions and use that structure to identify ways to rewrite and operate accurately with them. They can add, subtract, and multiply polynomials that extend beyond quadratics. They are able to rearrange formulas to highlight a quantity of interest, for example rearranging Ohm’s law, V = IR, to highlight resistance R. They are also able to create equations and inequalities representing relationships between quantities, including those that extend beyond equations or inequalities arising from linear, quadratic, and simple exponential functions to include those arising from simple rational functions. They are able to use these equations/inequalities to solve problems both algebraically and graphically. They can solve linear equations and inequalities; systems of linear equations; quadratic, simple rational, and radical equations in one variable; and recognize how and when extraneous solutions may arise.

Students prepared to exit this level also have a basic understanding of functions, can use function notation properly, and use such notation to write a function describing a relationship between two quantities. They are able to evaluate functions for inputs in their domains and interpret linear, quadratic, and exponential functions that arise in applications in terms of the context. They are able to construct, graph, compare, and interpret functions (including, but not limited to, linear, quadratic, and exponential). They can sketch graphs given a verbal description of the relationship and identify and interpret key features of the graphs of functions that arise in applications in a context. They are able to select or define a function that appropriately models a relationship and to compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

*Geometry:* Students prepared to exit this level can solve problems involving similarity and congruence criteria for triangles and use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. They can apply the concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU’s per cubic foot).

*Data Analysis and Statistics*: Students prepared to exit this level can summarize, represent, and interpret data based on two categorical and quantitative variables, including by using frequency tables. They can compare data sets by looking at commonalities and differences in shape, center, and spread. They can recognize possible associations and trends in data, in particular in linear models, and distinguish between correlation and causation. They interpret one- and two-variable data, including those with linear and non-linear relationships. They interpret the slope (rate of change) and intercept (constant term) for a line of best fit and in the context of the data. They understand and account for extreme points of data in their analysis and interpret relative frequencies (joint, marginal and conditional).

1. Center for Educational Assessment Research Report No. 913, Amherst, MA: School of Education, University of Massachusetts Amherst. This report was prepared as part of a contract with the Massachusetts Department of Elementary and Secondary Education’s Office of Adult and Community Learning Services. [↑](#footnote-ref-1)
2. *Measures and Methods for the National Reporting System for Adult Education* (ICR No. 1219.24] 1830-0027 [↑](#footnote-ref-2)
3. *Implementation Guidelines: Measures and Methods for the National Reporting System in Adult Education*, Division of Adult Education and Literacy, Office of Career, Technical & Adult Education, U.S. Dept. of Education, February 2016. [http://www.nrsweb.org/foundations/implementation\_guidelines.aspx](http://links.govdelivery.com/track?type=click&enid=ZWFzPTEmbWFpbGluZ2lkPTIwMTYwMjE2LjU1MjY2NDAxJm1lc3NhZ2VpZD1NREItUFJELUJVTC0yMDE2MDIxNi41NTI2NjQwMSZkYXRhYmFzZWlkPTEwMDEmc2VyaWFsPTE3NjEzOTk2JmVtYWlsaWQ9am9obi5sZW1hc3RlckBlZC5nb3YmdXNlcmlkPWpvaG4ubGVtYXN0ZXJAZWQuZ292JmZsPSZleHRyYT1NdWx0aXZhcmlhdGVJZD0mJiY=&&&100&&&http://www.nrsweb.org/foundations/implementation_guidelines.aspx?utm_content=&utm_medium=email&utm_name=&utm_source=govdelivery&utm_term=). [↑](#footnote-ref-3)
4. “The grades K–8 mathematics standards are organized by grade level, with four or five domains within each level. Under each domain are overarching standard statements followed by a cluster of related standards. For high school, the CCSS are organized by conceptual categories, which together portray a coherent view of high school mathematics and span traditional high school course boundaries. These conceptual categories include: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. Under each conceptual category there is an organizing structure similar to that used in K–8: domains with overarching standard statements, with each followed by a cluster of related standards” (Pimental, 2013, p. 46). [↑](#footnote-ref-4)
5. This appendix is Appendix E from the *Implementation Guidelines: Measures and Methods for the National Reporting System in Adult Education*, Division of Adult Education and Literacy, Office of Vocational and Adult Education, U.S. Department of Education, February 2016. Available at [http://www.nrsweb.org/foundations/implementation\_guidelines.aspx](http://links.govdelivery.com/track?type=click&enid=ZWFzPTEmbWFpbGluZ2lkPTIwMTYwMjE2LjU1MjY2NDAxJm1lc3NhZ2VpZD1NREItUFJELUJVTC0yMDE2MDIxNi41NTI2NjQwMSZkYXRhYmFzZWlkPTEwMDEmc2VyaWFsPTE3NjEzOTk2JmVtYWlsaWQ9am9obi5sZW1hc3RlckBlZC5nb3YmdXNlcmlkPWpvaG4ubGVtYXN0ZXJAZWQuZ292JmZsPSZleHRyYT1NdWx0aXZhcmlhdGVJZD0mJiY=&&&100&&&http://www.nrsweb.org/foundations/implementation_guidelines.aspx?utm_content=&utm_medium=email&utm_name=&utm_source=govdelivery&utm_term=). [↑](#footnote-ref-5)